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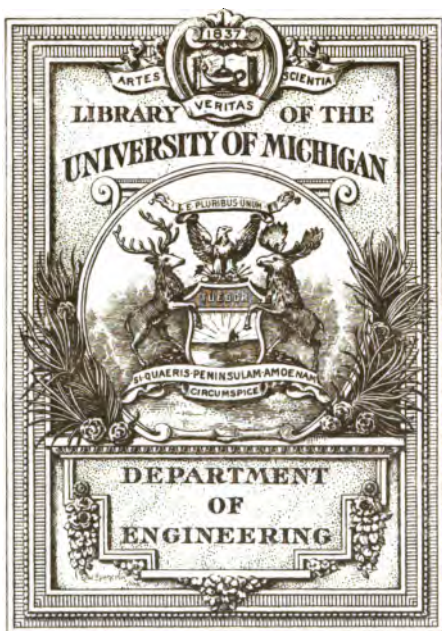
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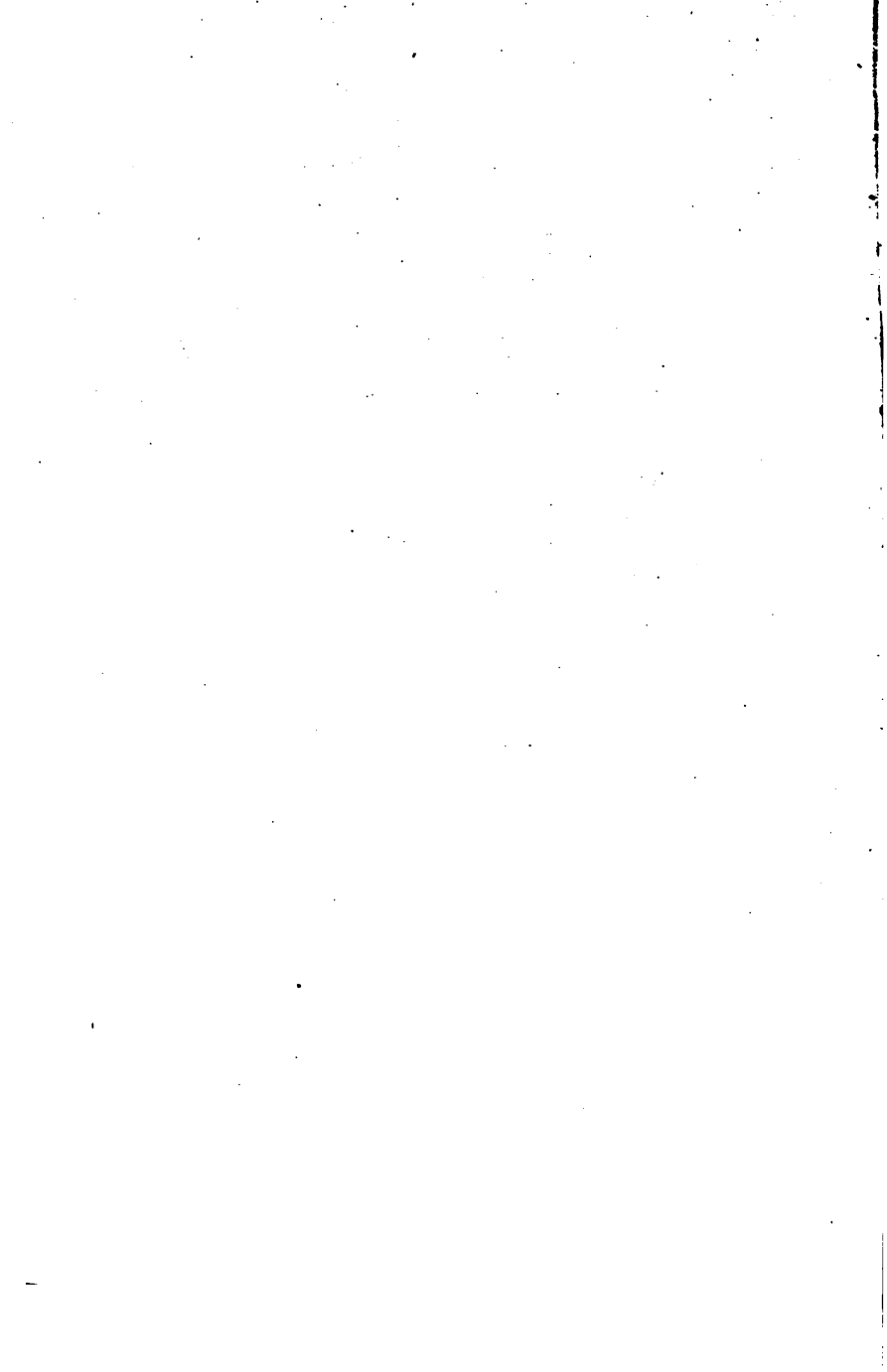
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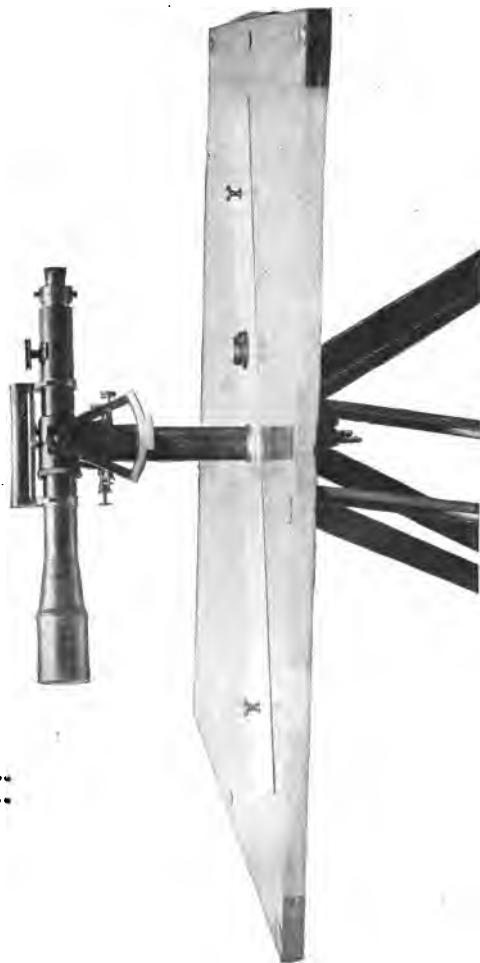
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Plane Table and Telescopic Alidade. (U. S. Geological Survey System.)

# THE PLANE TABLE

AND

## ITS USE IN SURVEYING

BY



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## SYMBOLS

|                                                                                     |                                    |
|-------------------------------------------------------------------------------------|------------------------------------|
|    | Triangulation point.               |
|    | Plane-table station.               |
|    | Church.                            |
|    | School house.                      |
|    | Factory.                           |
|    | House or shop.                     |
|    | Barn or shed.                      |
|    | Monument.                          |
|    | Windmill.                          |
|    | Signal.                            |
|    | Cemetery.                          |
|    | Fence Corner.                      |
|  | Fir Tree.                          |
|  | Deciduous Tree.                    |
|  | Dead Tree.                         |
|  | Hill tops and locations generally. |

# THE PLANE TABLE

AND

## Its Use In Surveying

### INTRODUCTION.

The plane table, one of the oldest of surveying instruments, is, in its simplest form, merely a board for holding the paper or other material upon which a map is drawn with the aid of a rule or straight edge.

Although the plane table has been known and used in Europe to a greater or less extent for more than three centuries, at the present day, in spite of its obvious advantages as regards speed, economy convenience and adaptability for surveying purposes, its use is limited mainly to government surveys in the mapping of large areas of country.

Although a useful and serviceable instrument for railroad and land surveyors, it has never come into general use by them in the United States.

Why this is so, is hard to explain, unless perhaps because of lack of knowledge of the instrument and its methods, as little has been written on the subject. Of late years it is gradually becoming better known. All recent text-books on surveying give some space to a description of its use and some instruction is given on the subject in many of the colleges and technical schools throughout the United States.

The plane table is used extensively in European countries as Germany, Austria, and Italy, etc. on government surveys and in the topographical survey of India by the British government.

In the United States it has been used for many years by both the Coast and Geological Surveys, and with the increasing demand for the mapping of parks, preserves, municipalities and various tracts of land for public or private purposes there is reason to believe it will gradually come more and more into use as surveyors learn to appreciate the advantages of the instrument.

## FORMS OF PLANE TABLES.

A plane-table outfit consists of a drawing-board, mounted on a tripod, to which is attached the map or sheet, upon which the work is to be plotted; a rule or straight edge, called an alidade, either with or without a telescope, used for sighting and drawing lines to objects it is desired to locate; a spirit-level; and sometimes, though not essential, a declinatoire or compass, to aid in approximate orientation of the table.

An umbrella mounted on a long handle with a sharp iron point to admit of its being firmly driven into the ground is often used to protect the plane table from the glare of the sun.

Numerous kinds of plane tables have been invented and used at different times, most of the variations being in the tripod head or movement.

This in most cases has been supplied with leveling screws, but in the Johnson and Gurley tables of recent years the screws have been dispensed with and a universal joint motion used for leveling the table.

The plane table movement or tripod head is made of brass and in the U. S. Coast Survey pattern three leveling screws are used, the horizontal motion is effected by two plates sliding upon each other, and a tangent screw is attached for slow movement in azimuth.

The Johnson plane table used by the U. S.

Geological Survey is much lighter, although sufficiently firm and rigid for all practical purposes.

In this movement there are no leveling screws, both vertical and horizontal movements of the table being effected by a ball and socket joint, and the tangent screw is dispensed with as the table can be moved in azimuth by hand with all sufficient accuracy. For triangulation the size of the board commonly used is 24 by 36 inches.

Folding tables for transportation on horse-back are sometimes used in rough country, and a sketch board made to serve as a plane-table in reconnaissance surveys for military purposes.

Of alidades there have been many patterns. The earliest type, a flat straight edge with raised sights, called a sight alidade, is still used in running lines of traverse and also to a limited extent in plane table triangulation.

The form of alidade used by the U. S. Geological Survey consists of a brass or steel rule or straight edge, 18 or 24 inches long, upon which the telescope is mounted on a standard four inches high, the magnifying power of the lens being about 20 diameters.

Telescopic alidades are provided with a striding level and a graduated vertical arc by means of which the altitudes of all located points may be determined, and in addition to the cross-hairs, two more horizontal hairs, called stadia-hairs, are sometimes added for use in stadia work.

## **ADJUSTMENTS OF THE ALIDADE.**

**Parallax:** Move the eye-piece until the cross-hairs are distinct and then sight some distant object. If there is no change in position of cross-hairs relative to the object when position of eye is altered, there is no parallax.

If position of cross-hairs varies, change focus of the eye-piece until both cross-hairs and object can be seen with distinctness and without alteration of position, or parallax.

**Collimation:** Place intersection of vertical and horizontal lines (cross-hairs) of telescope on some small but distinct object as for instance a nail-head. Revolve the telescope 180 degrees. If the intersection still covers the object sighted, the adjustment is correct. If not, correct one-half of the difference by moving the diaphragm by the adjusting screws, again sight the object, reverse the telescope, and, if there is still an error of collimation, correct one-half by the adjusting screws as before and repeat until adjustment is perfect.

**Striding level:** Place level on alidade and bring bubble to center by means of tangent-screw of the alidade. Reverse the level and if bubble remains in center the adjustment is perfect. If not, correct one-half by tangent-screw and the other half by adjusting screw attached to the level. These adjustments should be made at every station. .

Telescopic alidades sometimes have one or more levels attached to the rule or instead a circular level may be carried by the plane-table.

To adjust the attached levels, place the alidade in the center of the table drawing lines to indicate its position and bring the level bubble to the center by leveling the table. Reverse the alidade 180 degrees and if the bubble does not remain in the center, correct one-half the difference by adjusting the level screws and one-half by leveling the table.

These levels, however, are not essential as the striding level can be used instead for leveling the table, as well as for taking vertical angles.

The alidade rule should be an exact straight edge. To test its fiducial edge draw a line along it, reverse the alidade placing it exactly on the line and draw another line. If the two lines coincide the edge is true. To ascertain if the sides of the alidade rule are parallel they should be tested by sighting to the same point. If they are not parallel, care should be taken to draw lines on one side only.

Telescopic alidades are of various magnifying powers, say from 10 to 24 diameters.

High power is a disadvantage in hazy weather, as the particles of vapor or dust in the air are magnified correspondingly.

It might be supposed that high magnifying power would be an advantage, but this is not so, beyond a certain limit.

Only a given amount of light can enter the

object-glass and the greater the magnifying power the less clear will be the image.

Two eye-pieces, for high and low power, the former for clear and the latter for hazy weather would be a convenience.

The magnifying power of a telescope is the ratio of the focal length of the eye-piece to that of the object glass.

To ascertain the magnifying power, point the telescope towards the bright sky holding a slip of thin white paper before the eye-piece at the distance at which the image of the object-glass which is projected on it, is well defined.

Divide the diameter of the object glass by the diameter of the image.

For instance, if diameter of object glass is two inches and the image 0.1 inch the magnifying power would be  $2 \div 0.1 = 20$ .

## PLANE-TABLE TRIANGULATION.

The plane table is primarily an instrument for graphic triangulation.

Instead of reading and recording angles, and afterwards computing the resulting triangles, as with a transit or theodolite, the triangles are drawn directly upon the map, or projection, on the plane-table board, thus avoiding possible errors of record, adjustment and plotting. By this method much time is saved, the plane table being the most rapid and economical instrument for the purpose for which it is used, that has yet been devised. It has the additional advantage that any point can usually be tested from different stations in the field at time of survey and any error in its position detected, all plane-table locations being exact to scale employed for the map. It is especially adapted for use in country of considerable relief, or open rolling country, in fact, in all regions except heavily-wooded areas and plains and plateaus of low relief.

Although originally intended for triangulation, the plane table has other uses and properly equipped becomes well-nigh a universal surveying instrument, as, in addition to triangulation, it is used for traverse, topographic sketching and vertical angulation, in short, all the processes required for making a complete map.

In beginning plane-table triangulation a section

of country is usually selected in which primary triangulation stations have already been established.

The scale and area to be surveyed are decided upon, a spherical projection is made, the polyconic being used on nearly all U. S. Government maps, and upon this projection the primary triangulation points are plotted in their relative position in latitude and longitude.

These, if possible, should be at least three in number. A primary triangulation point is then occupied and the table leveled, either by the striding level attached to the alidade or a round level which can be permanently affixed to the alidade rule or carried in the pocket.

Place this level in center of table and bring the table to a level by the leveling screws, if any, or by adjusting the legs which can be easily done after a little practice. Unless the table is of more than usual evenness and solidity, it cannot be brought to an exact level anywhere away from the center, but, as is explained later, this is not necessary. After adjusting the alidade for parallax and collimation, the table is *oriented* by sighting some other triangulation point, that is, the projections of these points on the sheet or map upon which the triangulation is to be done, are brought into the same relative positions as the points themselves on the ground. Lines are then drawn to all objects in the landscape, such as signals, church spires, cupolas, chimneys, flag-poles, prominent

trees, hill-tops, spurs, etc., that may be of use in obtaining proper control for the resulting map. Tangents should also be drawn to all railroads, highways, shore lines and other objects in the landscape that it may not be possible to locate by intersection.

Lines drawn to signals and other points available for plane-table stations are called *foresights*.

The points to which foresights have been drawn are occupied and the table again oriented by placing the alidade on the foresight and sighting back to station from which foresight was taken.

Another located point is then observed and the position of point occupied determined by *resection*.

A third located point is then sighted and, if the line from this passes through intersection of the lines from the other two points, the table is in position and location of station correctly determined.

All objects to which lines were drawn from the first station that can be identified, are sighted and located by *intersection* and lines drawn to other objects that come into view as well, and in this way by the use of foresights an entire sheet may be plane-tabled, but it usually happens sooner or later that resort must be had to the three-point method of location.

1. It may occur that none of the triangulation points are accessible and in that case it will be necessary to begin plane-tabling with a three-point station.

2. It may be found on occupying a point to



which a foresight has been drawn that lines from other located points will not intersect, but form a triangle,—*the triangle of error*—caused by an error in the foresight or in identification of point to which foresight was drawn. In that case the table is out of position and must be brought into position by the three-point method.

3. In a country of great relief where the triangulation points are at a considerable elevation above the valleys it will often be found very difficult to recognize objects to which lines have been drawn. In that case it is often better and more convenient to make three-point stations in the valleys or on the lower hills, the commanding positions of the triangulation points making them easily discernible with the additional advantage that objects to which lines have been drawn can more easily be identified.

4. Much time may be saved by the use of three-point stations as fewer signals are necessary, and more stations can be made and more points located in a given time.

### THE THREE-POINT PROBLEM.

The three-point problem presents no difficulties, and no one can be considered thoroughly competent for plane-table work who does not understand it.

The rules to be remembered are few and simple. In making a three-point station the plane table is set up in such position that at least three previously located points are visible and in such relation to each other and the point to be determined that lines drawn from them will intersect at large angles, say between  $30^{\circ}$  and  $120^{\circ}$ .

The position of the point sought in relation to the three located points from which position of the former is to be determined may be (Fig. 1).

1. Inside the triangle formed by the three located points.
2. Outside the triangle, but inside the great circle (the imaginary circle passing through the three points on the ground).
3. On the great circle.
4. Outside the great circle.

In all cases the point sought is on the same side of line drawn from each located point, that is, if on the right side of one line looking toward the point from which the line is drawn, it is on the right side of the other lines, and distant from the lines drawn from the three points in proportion to its distance from each point.

When the plane table is not in position lines drawn from the fixed points instead of intersecting at one point will form a triangle called the triangle of error, unless the point occupied should chance to be on the circle passing through the three fixed points, in which case the location of the point occupied is indeterminate.

1. If point sought is within the triangle on the

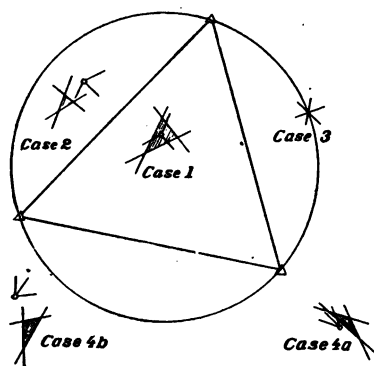


FIG. 1.

ground, in other words, the great triangle, the true point is within the triangle of error and distant from the lines from the three located points in proportion to their distances from the point occupied.

2. When the point sought lies within either of the three segments of the great circle formed by the sides of the great triangle, the true point is without the triangle of error and the line drawn

from the middle point lies between the true point and the intersection of the lines from the other two points.

3. When the point sought is on, or very near the great circle, the position is indeterminate.

4a. When the point sought is without the great circle and the middle point is on the near side of the line joining the other two points, as is the case when the point lies inside of one of the angles formed by the sides of the great triangle produced, the true point is without the triangle of error and the line drawn from the middle point lies between the true point and the intersection of the other two lines.

4b. When the point sought is without the great circle and the middle point is on the far side of the line joining the other two points, the true point is without the triangle of error, and on the same side of the line from the middle point as the intersection of the other two lines.

The following rule applies to all points outside of the great circle: The point sought is always on the same side of the line from the most distant point as the intersection of the other two lines and distant from each line in proportion to the distance of the point from which it was drawn. In case, as occasionally happens, it is difficult to decide which of the three points is the most distant, the relation of the line from the middle point to the lines from the other two points as described in 4a and 4b will determine on which side of the triangle of error the point sought lies.

In the rare case where the three located points are in a straight line the same rule applies as in the case 4a, that is, the line from the middle point lies between the position of point sought and the intersection of the lines from the other two points.

The above rules for solving the three-point problem form what is commonly called the method by *trial* and is the one usually employed in the field by practical plane-tables.

Other methods will be described later

In making a three point station, if the lines drawn from the three fixed points form a triangle of error, showing that the table is not in position, place the alidade on the most distant point, and over the point on the paper, inside or outside of the triangle of error, as the case may be, which as near as can be judged is the location of the point sought. Unclamp the table, turn it in azimuth until the far point is sighted. Clamp the table again, and draw lines from the three fixed points. If these lines intersect in a common point the table is in position. If not, repeat the process, until the intersection is perfect. In short, *orient* on the most distant point, *resect* on the nearer points.

The reason for orienting on the most distant point is that any movement of the plane table changes this point more in azimuth than the nearer points, while a slight turning of the table would have but little effect in the position of a nearby point.

However, in case a foresight has been taken to

or near the point occupied, it is well to orient by this foresight and resect from the other two points.

If a triangle of error is formed it will usually be small, and then the rules applicable to the three-point problem can be followed.

After the three point station is made, that is, accurately located, the plane-tableer should sight a fourth well located point such as a signal, lone tree, church spire, etc., if any are visible, as a check on the position, and if a line from this point passes through the station occupied making a good intersection, (an angle between 30 and 120 degrees) the station may be considered well determined. Other located points may also be sighted as a test of the accuracy of the work.

Sometimes in making a three-point station a mistake in position of one or more of the located points, from which the station is to be determined, may be made; that is, a point nearby may be mistaken for the signal at the located point.

In this case it is well to remember that a three-point station can apparently be made, that is, lines from the three points will intersect without forming a triangle of error, although the location on the map will be wrong, and if the true point happens to be very near the point which is mistaken for it, the error in location of the three-point station cannot be detected unless a fourth located point, not on the great circle, is visible which may be used to test the location of the station.

In short, the fact that lines supposed to be drawn from three located points intersect without a triangle of error, is no proof of correct location of station, as any three lines will *come in*.

Although stations located in the manner may be but slightly out of position, points cut in from such stations may be greatly in error.

However, if the three points are well located and there is no doubt about their identification, a well determined three-point station can be made from them, and afterwards when still another point has been located from the three initial points the accuracy of the work can be tested as although any three points will come in, lines from four points can not meet in the same intersection, unless they are correctly located in reference to each other. Although they may appear to do so, if two of the points lie in nearly the same direction from the station occupied. In the selection of a position for a three-point station, care should be taken that the point chosen, if inside the triangle, is not nearly in line between two of the three points, or, if outside the triangle, is not on nor very near the great circle, nor so far outside, that lines from the three points intersect at such angles, less than 25 or 30 degrees, that the intersection is indefinite, unless a fourth point is discovered that will serve as a check.

Sometimes the plane-table may find himself in a region of limited view where only short sights can be obtained. In this case, when orienting

from a short foresight, great care must be taken to sight back to the exact point from which the foresight was taken, some slender object as a pole having been left there to indicate the precise spot occupied, and the points that are used for resection should also be small and sighted with extreme care.

If great care is not exercised in this respect, on developing the triangulation until some distant point before located is sighted, it will probably fail to come in, that is, the location of the point occupied, will be found to be in error, although the locations made by the short sights were apparently correct in themselves.

This illustrates the extreme hazard of carrying on a system of plane-table triangulation, based entirely on three points, and emphasizes the necessity of always having four well located and distributed points in view from every station, if possible.

In plane-table work, as in primary triangulation a system of quadrilaterals is to be preferred to that composed of plane triangles. In making three-point stations a compass will be found a convenience in orienting the table approximately.

In conjunction with the stadia or telemeter, the plane-table may be used in locating nearby points by radiation, so called.

In this manner with one or more rodmen a large number of points can often be located from one station.

It is a most convenient and satisfactory method

for the survey of lakes and rivers, which are often so situated as to be very difficult or impossible to locate by triangulation. The stadia-rod can also be advantageously used in a survey on a large scale, where it is desirable to locate many points within a small area.

To make a stadia-rod, take a strip of smooth unpainted board, from four to six inches wide, and ten to fifteen feet long. Mark off the board into sections six inches in length and paint each alternate section black.

Set up the plane table and sight the rod removing it to such distance that two adjacent stadia-hairs will cover one division on the rod. Measure distance between telescope and rod carefully, with tape or chain. In most telescopes the stadia-hairs intercept one six-inch division on the rod for every hundred feet. If this should not prove to be the case, whatever distance is intercepted should be taken for the unit of distance. For more exact work stadia-rods can be procured of makers of engineering instruments graduated to any required fineness.

To locate a station by the tracing paper method attach the paper to the board and mark a point upon the paper or cloth for the point sought. From this point sight and draw lines to the three known points from which the station is to be determined. Then shift the tracing paper until each of the three lines passes through the plotted point corresponding to the point toward which it was drawn. Posi-

tion of the point sought will be at the intersection of these lines, which can be pricked through the paper on to the plane-table sheet. This method is quite often used, but is apt to be inaccurate unless great pains are taken in drawing the lines to the sighted points, and is impracticable in windy weather.

The three-point problem may be solved geometrically with the aid of compasses as follows:

Let  $a$ ,  $b$ , and  $c$  be the projections on the plane-table sheet of the three located stations on the ground from which the point occupied is to be determined.

Draw a circle through  $a$ ,  $b$  and the intersection of the lines from these points which form one angle of the triangle of error; also draw a circle through  $b$ ,  $c$  and the corresponding intersection. A third circle, as a check on the work can be drawn through  $a$ ,  $c$  and the intersection of the lines from these points.

The intersection of these circles is the location of the true point.

Place the alidade on this point, orient, or bring table into position, by sighting one of the fixed points and verify the position by resecting on the other located points. This method is inconvenient in the field, but is useful in the office as an aid to the proper understanding of the three-point problem.

*Bessel's method by inscribed quadrilateral*, or the exact method so-called, another solution of the



with the alidade on  $c$ , sight the middle point  $B$  and draw a line  $ce$  along the edge of the alidade rule. Then, with the alidade on the line  $ac$  revolve the table until the point  $C$  is sighted.

Clamp the table, place the alidade on  $a$ , sight the middle point  $B$  and draw the line  $ae$  along the edge of the rule. A line drawn through  $e$  and the middle point  $b$  will pass through the point sought ( $d$ ).

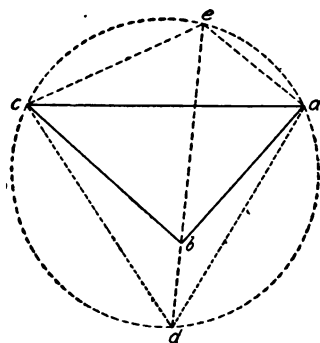


FIG. 3.

Set the alidade on this line, revolve table until point  $B$  is sighted and table will be in position.

Place the alidade on  $a$  and direct to  $A$  and draw a line along the alidade rule.

The intersection of this line with the line  $be$  is location of point sought.

Verify the position by placing the alidade on  $c$  and resecting on  $C$ .

The following analytic solution of the three-point problem is adapted from Chambers' Practical Mathematics.

Given the distances between any three points and the angles subtended by them at a station to find the relative position of the station and its distance from each of the three points.

CASE I.—When the station is outside the triangle

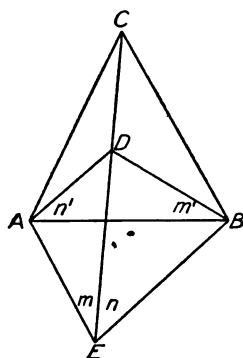


FIG. 4.

formed by lines joining the given points and the middle point is beyond the line joining the other two points (Fig. 4).

Let  $A, B, C$  be the three points,  $E$  the station occupied and  $m, n$  the angles read from  $E$ .

Make the angle  $A, B, D = m'$  and the angle  $DAB = n'$ .

In the triangle  $ABC$  the three sides are given hence angle  $A$  can be found. In the triangle

$A'DB$ , the angles and side  $AB$  are given; hence  $AD$  can be found.

In triangle  $ADC$ ,  $AC$  and  $AD$  are given, and angle  $A = CAB - DAB$ ; consequently angle  $ACD$  can be found.

In triangle  $ACE$  the angles and side  $AC$  are known; therefore the sides  $AE$  and  $CE$  can be found.

Then in triangle  $ABE$  the sides  $AB$ ,  $AE$  and

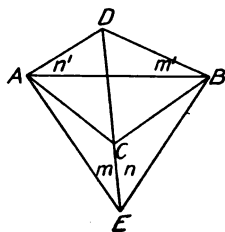


FIG. 5.

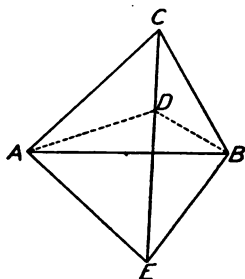


FIG. 6.

the angles being known the side  $BE$  can be obtained.

CASE II.—When the station occupied is without the triangle, and the middle point is on the near side of the line joining the other two points (Fig. 5).

Let the middle point  $C$  be between the station  $E$  and the line  $AB$  then the point  $E$  and  $D$  will be both without the triangle  $ABC$  and on opposite sides of it, and the solution will be analogous to that of the first case.

CASE III.—When the station is within the triangle (Fig. 6).

Let  $D$  be the station then, the angles  $A D C$ ,  $B D C$  being given their supplements are known.

Make angles  $A B E$ ,  $B A E$ , respectively =  $A D E$  and  $B D E$ .

$$\text{Angle } C A E = C A B + E A B.$$

$$\text{Angle } C A D = 180^\circ - (A C D + A D C) \text{ and}$$

$$\text{angle } B A D = C A B - C A D.$$

In the triangle  $A C D$  the angles  $C A D$  and  $A D C$  are known and the side  $C$ , from which the sides  $A D$  and  $C D$  can be found.

In the triangle  $B C D$  the angle  $C$  and the sides  $B C$  and  $C D$  are known from which the remaining side  $B D$  can be found.

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\* An original and ingenious graphic solution of the three-point problem is given by Prof. Llano in the Engineering News for December 29, 1904.

### THE TWO-POINT PROBLEM.

It may sometimes be desirable to place the table in position at a point from which only two located inaccessible points can be seen.

The following solution of this problem is taken from the U. S. Coast and Geodetic Survey Report for 1897-98:—Fig. 7.

To put the plane table in position at a third point  $C$  by resect on from two located points  $A$  and  $B$ , whose projections on the sheet are represented by  $a$  and  $b$ , select a fourth point  $D$  so that intersections from  $C$  and  $D$  upon  $A$  and  $B$  will make angles sufficiently large for good determinations.

Put the table approximately in position at  $D$ , by estimation or compass, and draw lines  $Aa$ ,  $Bb$ , intersecting in  $d$ . Through  $d$  draw a line directed to  $C$ . Then set up at  $C$ , and assuming the point  $c$  on the line  $dC$  at an estimated distance from  $d$ , and putting the table in a position parallel to that which is occupied at  $D$ , by means of the line  $cd$ , draw lines from  $c$  to  $A$  and from  $c$  to  $B$ .

These will intersect the lines  $dA$ ,  $dB$  at points  $a'$  and  $b'$ , which form with  $c$  and  $d$  a quadrilateral similar to the true one, but erroneous in size and position. The angles which  $ab$  and  $a'b'$  make with each other is the error in position.

By constructing through  $c$  a line  $cd'$  making the same angle with  $cd$  as that which  $ab$  makes with

$a' b'$ , and directing this line  $c d'$  to  $D$ , the table will be brought into position and the true point  $c$  can be found by the intersections of  $a A$  and  $b B$ .

Instead of constructing with drawing instruments, the angle of error in the position of the table, that is the angle of the line  $a' b'$  makes with the line  $a b$ , which is not always convenient, the following expedient may be adopted.

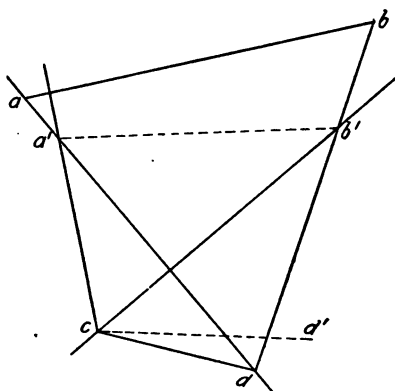


FIG. 7.

$a' b'$  is now parallel to  $A B$  and to bring the table into position it must be turned until  $a b$  is parallel to  $A B$ .

To do this set up a pole or other mark in the direction  $a' b'$ ; set alidade on  $a b$  and revolve the plane table until  $a b$  points to the mark.

Then  $a b$  is parallel to  $A B$  and the table is oriented.

Another solution of the two-point problem is as follows (Fig. 8):

After the table has been placed in position by estimation, resect upon  $A$  and  $B$ , the two lines intersecting at  $c$ . The angle  $a b c$  is the angle subtended by  $A B$  at  $C$  and the position of the point occupied ( $C$ ) must be on the circle passing through  $a b c$ .

Select a fourth point  $D$  nearly at right angles

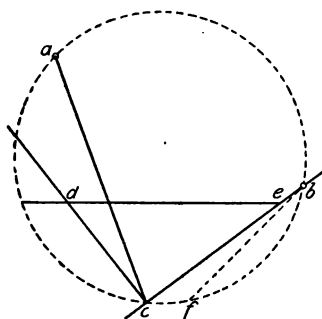


FIG. 8.

to  $b c$ . Sight the signal  $D$  and draw line  $c d$ .

Occupy  $D$ , place the alidade on the line  $c d$  and sight  $C$ , thereby bringing the table into a position parallel to its position when at  $C$ . With the alidade on  $d$  observe the signal at  $B$  and draw the line  $d e$  intersecting  $c b$ .  $c e$  is the distance of  $C$  from  $B$ . Lay this distance off from  $b$  in the direction of  $c$  as a chord of the circle drawn through  $a b c$ .

The intersection of the chord with the circle at  $f$  is the true location of station  $C$ .

The two point problem can also be solved with tracing paper as follows:

The two located points  $A$  and  $B$  are plotted on the plane-table sheet as  $a$  and  $b$  (Fig. 7). It is desired to find the position of a third point  $C$ . Occupy a fourth point  $D$ , so placed that lines drawn from the other points will intersect at sufficiently large angles. Orient the table by estimation or with the compass.

On the tracing paper which is fastened to the board, indicate a point to represent the location of  $D$ . Draw lines from this point towards the stations  $A$   $B$  and  $C$ . Lay off the line  $dc$  the estimated distance to scale between  $d$  and  $c$ . Occupy the station  $C$  and bring the table into a position parallel to that at  $D$  by sighting back to  $D$  on the line  $cd$ .

Draw lines from  $c$  to  $A$  and  $B$ , intersecting the lines drawn to the same points from  $D$ , in  $a'$  and  $b'$ . The angle which the lines  $ab$  and  $a'b'$  make with each other is the error in position of the table. Move the tracing paper until the line  $a'b'$  is brought over the line  $ab$ , thereby bringing the table into position. Sight and draw a line from, in other words resect on  $A$  and  $B$ . The intersection of these lines will give the position of  $C$  which can be pricked through with a needle to the sheet.

When possible to get in line with two located points a two-point station may be made in the following manner. Set the plane table up any-

where in line with, and bring into position by sighting on, the located points. Sight the point where a station is to be made, which should be at some point affording angles large enough for good intersections, drawing the line at the most convenient place on the sheet.

Occupy the point to which the line is drawn, and bring the table into position by placing the alidade on the line and sighting back to the point just left from which the line was drawn.

The table will now be in orientation and the position of the station can be determined by resection on the two located points. Frequently stations can be made by *ranging in* between, or *lining up*, that is, getting in line outside of two located points. A foresight may be taken over a chimney, or a tree on a hillside, and a station made on the hilltop back of the same.

### **CENTERING THE PLANE TABLE OVER THE STATION.**

In regard to placing the plane table directly over a primary triangulation station, or other located point to be used as a station, it is well to remember that exact centering of the table is only necessary on maps of very large scale.

For instance on scale of 1 : 45,000, 75 feet on the ground is represented by  $1/50$  of an inch on the map and as  $1/150$ , or at the extreme  $1/200$ , of an inch is about the limit of human vision, if the plane-table is set within few feet of the exact point there can be no appreciable error.

Plane-tablers have sometimes taken a great deal of time and trouble to remove and afterwards replace a signal in order to set directly over the station when the scale of the map was so small as to render such care entirely unnecessary.

In regard to errors arising from imperfect leveling of table, it has been shown by Josiah Pierce in his essay on the "Economic Use of the Plane table," that errors from this cause amount to practically nothing unless the inclination of the board is very great.

A plane table may be 15 degrees out of level before the maximum error in the measurement of a horizontal angle will amount to one degree.

## VERTICAL ANGULATION.

The process of obtaining altitudes by vertical or dip angles is peculiarly adapted to the plane table. The vertical angle between the station occupied and any located point, of which it is desired to ascertain the elevation, is read, the distance between the two points is carefully measured on the map and from this data, after the correction for curvature and refraction is made, the difference in height of the two stations is determined.

Differences in altitude or elevation may be computed from a table of natural tangents, but, as this is a rather tedious operation when a large number of angles are to be calculated, vertical angle tables adapted to the scale of the map and containing corrections for curvature and refraction are generally used.

Every point whose elevation it is desired to ascertain with any degree of precision should be observed from two or more stations whose heights are well determined, and the stations from which elevations of other points are to be obtained should be determined by a series of vertical angles from two or more stations. Reciprocal angles are not checks and elevations should not depend upon them alone.

Any datum can be assumed, but if it is desired to connect the results obtained from vertical angles

with any special datum, as sea-level, a line of levels can be run from some convenient bench mark to one or more signals or other exact and well determined point on the map.

To obtain the best results with vertical angles exact points must be sighted, as for instance a cross-piece attached to a signal, the height of which above the ground is known.

The height of the telescope above the station occupied must also be measured. This will be found to be ordinarily about 4.5 feet.

The correction for curvature of the earth is 0.667 of a foot for the first mile, increasing as the square of the distance, or two-thirds the square of the distance in miles equals the curvature in feet.

The usual formula for computing the combined curvature and refraction is 0.574 into square of distance in miles equals curvature and refraction in feet. In making allowance for refraction it is usually reckoned as one-seventh of the curvature, but may vary greatly from this, although for distances of 8 or 10 miles the variation of the refraction would not be the cause of any large error that might occur.

Large errors in vertical angles at these distances are caused by too coarse or imperfect graduation of the vertical arc, imperfect adjustment of alidade, difficulty in leveling the alidade because of windy weather, error in location of point, or mistakes in reading the angles. With an arc reading to one minute, and by estimation to one-half minute,

at a distance of 5 miles, angles taken to exact points as signals, lone trees or houses should check out within 5 feet under favorable conditions of weather. Up to distances of seven or eight miles, elevations of even wooded summits should be obtained with no greater error than half a 20 ft. contour interval, say ten feet. In careful vertical angle work it is well after reading an angle to invert the telescope and observe again, taking the mean of the observations and thereby eliminating the errors of adjustment.

The following illustration taken from the U. S. Coast and Geodetic Survey Report for 1880 will best show how this is done:

Telescope direct:

|                            |           |           |
|----------------------------|-----------|-----------|
| Level direct, reading..... | +0° 1'    |           |
| Level reversed.....        | 0'        |           |
|                            | <hr/>     |           |
| Mean.....                  | +0° 0' .5 |           |
| Angle to point.....        | +2° 17'   |           |
|                            | <hr/>     |           |
| Elevation (difference).... |           | 2° 16' .5 |

Telescope inverted:

|                            |          |           |
|----------------------------|----------|-----------|
| Level direct, reading..... | 0° 2'    |           |
| Level reversed.....        | 1'       |           |
|                            | <hr/>    |           |
| Mean.....                  | 0° 1' .5 |           |
| Point.....                 | +2° 12'  |           |
|                            | <hr/>    |           |
| Elevation (difference).... |          | 2° 13' .5 |
| Mean.....                  |          | 2° 15'    |

It will be seen that the level was one-half minute out of adjustment, the horizontal wire one and one-half minutes, and that revolving the telescope about itself changed its relation to the index on the vernier by  $1'$ .

The mean is free from all errors of adjustment.

Before reading the vertical angles the adjustment of the striding-level should be tested, and the index-error of the vertical arc, if any, noted. If the zeros on the arc and vernier coincide when the telescope is level, there is no index-error. If they do not, the reading must be noted and the correction applied to the angles.

Most alidades of recent make have an adjustable vernier thus eliminating the index-error.

An example of a vertical angle computation follows:

Elevation of station above sea 250 ft. Distance of summit, whose height is to be obtained, 1.5 miles. Vertical angle  $+1^{\circ} 50'$ . 1.5 miles = 7920 ft. Tangent of  $1^{\circ} 50' = 0.03201$ .  $7920 \times 0.03201 = 253.5$  ft. 1.3 = correction for curvature and refraction and 4.5 ft.-height of telescope above ground at station.

$$253.5 + 1.3 + 4.5 = 259.3 \text{ ft.}$$

$$250 + 259.3 = 509.3 \text{ ft. height of summit.}$$

Angles of elevation are called  $+$  angles, of depression,  $-$  angles.

Although, as before stated, refraction is usually estimated as one-seventh of the curvature, it

varies greatly to an indefinite amount, increasing with the distance between the stations observed.

The best time for measuring vertical angles is between 9 a.m. and 3 p.m., as between these hours the vertical refraction is less variable than earlier or later in the day.

A book should be used in which to record vertical angles as well as such descriptions of the foresights, lines, and locations as may be desirable as an aid to identification, but at the same time it is well to put everything on the plane-table sheet, as numbers, graphic sketches, and written descriptions, that will tend to make the work legible to others than the plane-tabler in case of loss of record book, or the plane-tabling were to be completed by another. The Roman numerals may be used for numbering stations, the Arabic figures for foresights, locations, etc., and the altitudes should be placed on the sheet in red.

## SIGNALS.

Unbleached cotton cloth is best for signal flags. It wears better and is bleached by exposure to the air in a few days. Flags should not be over five or six feet in length, as longer flags will not blow out in light winds. If a large flag is desired, put up two flags, one under the other.

White is better than black or colored flags.

Color can not be distinguished at a distance of a few miles and a black flag will not show plainly, if at all, against a dark background, while a white flag appears black against a bright sky.

The signal pole may be from fifteen to forty feet in length, according to circumstances, but should at least rise high enough above the ground to allow the flag to blow out freely, and may be nailed to a tree, or fence, or set firmly in the ground, 2.5 to 3 feet, by tamping, as on occupying the point with the plane table it will not be necessary to remove the signal unless the work is on a very large scale as perfectly accurate results can be obtained by setting the table by the side of and within a few feet of the pole. On ledges or rocky hills props may be necessary to support the signal pole. These should be eight feet or more in length according to height of pole and at least three in number, four are preferable, set 120 degrees apart and nailed to the pole at different heights, a foot apart if possible, to prevent the pole "kick-

ing out " in high winds. The lower ends of the props should be driven firmly into the ground or else weighted down and held in place by heavy stones.

## LAND SURVEYS.

The plane table is well adapted to the survey of large or small tracts of land, as farms, parks, et cetera, on any scale desired, except in heavily wooded areas, and in many ways is more convenient and efficient than the transit or jacob-staff, as the work is plotted directly upon the sheet in the field at time of survey, thereby saving possible errors of recording and plotting.

In beginning the survey of a tract of land of which a map or plat is to be made, two intervisible points as nearly as possible on a level should be selected and the distance between them measured by chain or tape.

This distance will serve as a base-line. These points should be plotted on the map or plan at such distance apart as is required by the scale. Signals should be erected at each end of the base line and also at as many other prominent points as is desirable to insure proper control.

Set up the table at one end of base line, place alidade on the plotted points on the sheet which represent the stations at end of base-line, and revolve table until signal at other end of base-line is sighted. The table is now in position.

Sight and draw lines along the edge of the alidade rule to all the visible signals as well as other prominent points it may be desired to locate as trees, fence corners, houses, sheds, etc.

Then occupy the other end of base line and after bringing the table into the same position as at first station by sighting back to same, draw lines to all visible points to which lines were drawn from first station, whose positions are such that the intersections will form sufficiently large angles for good determinations. After completing work at this station, occupy a third point to which a line or foresight has been drawn and locate same by resection as it is called, that is, by sighting the first station occupied at end of base line.

From this last station other points can be intersected and lines drawn to new points that come into view.

In this manner the work can be carried on until the entire area is surveyed.

The position of points that can not be located by intersection, not being visible from any two stations, may be found by stadia, tape, or chain measurements, or even by pacing if the distance is not great. At every station, sight all previous stations and locations, as each location should be sighted from at least three stations as a check upon the accuracy of the work.

Altitudes can be determined by vertical angles, if the alidade has a vertical arc; and the configuration of the ground shown by contour lines drawn at any desired vertical interval.

In surveys on a large scale care must be taken to set the plane table over the exact point representing the station. In government surveys the

locations of primary triangulation points are usually indicated by a copper bolt, metal tablet, or iron or stone post; but, for temporary purposes, a chisel mark on a rock, or a wooden pin, is sufficient.

On a scale of 100 feet to 1 inch, 0.01 of an inch on the map represents 1 foot on the ground. In such cases to insure accuracy in the plane-table work, it is necessary to place the point on the table, corresponding to the station on the ground, very nearly over the pin marking the station, and, in order to do this properly, a plumb-line should be used and the table carefully plumbed over the station.

On maps of large scale the signal poles should be placed directly over the pin marking the point on ground and great care taken in sighting and drawing lines to signals and other objects so that the exact point may be intersected from each station.

The larger the scale, the more care is required in this respect.

In order that the plane table may be placed directly over the station, signals should be built so that they can be easily removed and replaced, or else the signal pole placed high enough from the ground to clear the head of the operator.

The magnetic meridian should be drawn on the sheet and if desired the true meridian can be obtained by a north star observation.

### PLANE-TABLE TRAVERSE.

On the U. S. Geological Survey the plane table is used largely as a traverse instrument.

That is, in ~~sa~~urveying or traversing roads, trails, streams, and valley ways, the table being oriented by compass, and distances obtained by wheel revolutions, stadia, tape, or pacing.

Used in this way, it is the most satisfactory and expeditious method of filling in the details of a map.

For this purpose a smaller and lighter plane-table is used with a sight alidade made especially for the purpose.

This alidade is a brass rule six inches in length and fitted with a front and back sight instead of a telescope, the short distances measured in traverse rendering a magnifying glass unnecessary.

## PROJECTIONS.

A map projection is an approximately rectangular diagram on which are plotted the parallels of latitude and meridians of longitude according to the scale of the map. Many different projections have been invented and employed from time to time, but the polyconic is considered the most satisfactory, as it involves less distortion of the earth's surface, and is the projection now used by all U. S. Government Surveys.

The U. S. Coast and Geodetic Survey publishes tables of this projection which can be used for any scale, and

Bulletin No. 234 of the U. S. Geological Survey contains tables adapted to several different scales with directions for laying-off or plotting the projection.

The paper used for the field sheet must be of the very best quality as it is a substance very susceptible to changes of weather, nor can fine delicate lines be drawn on poor paper.

Even the best paper of ordinary thickness has been found to be subject to considerable distortion from expansion and contraction caused by changes in the weather and to obviate this, two sheets of heavy paragon paper, so arranged that the grain of the sheets is at right angles, are pasted together with muslin between.

It has been found that the slight changes that

occur in paper prepared in this manner are distributed uniformly in all directions, thus reducing the distortion to a negligible quantity.

Celluloid sheets are sometimes used in very rainy regions, but with proper care double-mounted paragon eggshell paper can be used in almost any climate with satisfactory results.

Sheets of this double thick paper can not be rolled, but must be transported in flat tin or wooden boxes, or while in use in the field permanently attached to the plane-table board, both when not in use being kept in a wooden or sole-leather case.

Single thick paragon paper, properly seasoned, can be used for plane-table work with good results, if care is taken to protect it from the weather, and the field work does not take a great length of time.

Various appliances have been used, for fastening the plane-table sheet, or map, to the board, as clamps, screws, and thumbtacks.

Clamps are objectionable as they take up considerable space on the board, thereby interfering with the use of the alidade, are liable to slip, and do not hold the paper firmly in place.

Decidedly the most satisfactory contrivance for this purpose is a brass, flat-topped screw which is screwed by hand into a cylindrical brass screw, with an inside thread, set permanently into the board, flush with the surface of the table. This arrangement holds the paper firmly in position and is in every way satisfactory.

## CONCLUSION.

Two pencils should be used in plane-table work, one with a chisel edge for drawing lines along the edge of the alidade rule, and the other with a cone shaped point for drawing symbols, numbers, etc., on the map. These pencils should be very hard, 6 or 7 H at least and of the best quality:

For recording vertical angles etc., a softer pencil may be used. Sandpaper pads for sharpening the pencils and rubber pencil tips for erasing pencil marks are a great convenience.

A fine needle with a large head made by melting sealing wax, should be used for marking with a small needle-hole the exact intersection of the lines determining the position of a station or other located point.

Before beginning the plane-table work, it is a good plan to ride over a considerable portion of the area to be surveyed in order to get a general idea of the "lay of the land," position and distribution of the commanding points, and amount of control needed.

During this reconnaissance the main stations can be selected and signals erected, leaving minor stations to be chosen as occasion arises during the prosecution of the work.

In this way the progress of the work can be facilitated and an estimate made of the time required to complete the triangulation.

Sometimes it will be found necessary to return to a station a second, or even a third time, because of details at first overlooked or observations prevented by bad weather.

The amount of control, that is, the number and distribution of the stations, intersections and other located points, necessary to insure the required accuracy in the map, varies according to the character of the land surface and amount of detail, and must largely be left to the judgment of the plane-tableer; but, in general, it may be said, if the plane-table locations are properly distributed, and in sufficient number to insure that all details are shown without appreciable error to scale, the map is practically perfect.

One great advantage the plane table possesses over other surveying instruments is, that the map is made directly in the field and at every station all visible points previously located can be tested and any error discovered and corrected at once, and not as sometimes happens in transit work, remain undetected until long after when the angle is plotted in the office, where it is often impossible to rectify the error.

In short the plane-table method is the most rapid, economical and satisfactory way of making a map yet discovered. It is accurate to scale, detects and corrects error during the continuance of the field work and as before remarked is well-nigh a universal surveying instrument for all the purposes for which a map is intended, as has been

thoroughly proven by many years use on the Government surveys in all kinds of country.

Mr. Gannett, Geographer, U. S. Geological Survey, in his manual of Topographic Methods says, "For making a map the plane table is a universal instrument. It is applicable to all kinds of country to all methods of work, and to all scales. For making a map, it is the most simple, direct and economic instrument; its use render possible the making of the map directly from the country as copy, and renders unnecessary the taking of elaborate notes, sketches, photographs, etc., which is not only more expensive, but produces inferior results."

In conclusion, the following extract from Pierce on the "Use of the Plane Table" well expresses its merits as a surveying instrument:

"Recent results and improvements show, beyond a doubt, that a far greater degree of precision can be obtained from simple graphic triangulation than is commonly supposed, amply sufficient for purposes of mapping, and with a degree of economy unequalled by other systems.

"The engineer, or student, can not afford to lose sight of a valuable and simple instrument, with which he can, even alone and unassisted, construct a topographical map of any scale, with a degree of precision only limited by the scale, and in the same time that he would occupy in taking observations with other instruments"



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